MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

EME4076 – MECHANICAL VIBRATIONS

(ME)

24 OCTOBER 2018
02:30 p.m. – 04:30 p.m.
2 Hours

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 9 pages including cover page with 5 Questions and 3 appendices.

2. Attempt **FOUR** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1(i) shows a lighting fixture attached to the ceiling of a power plant at equilibrium. The length of the cable of the fixture is l and the mass of the bulb and its bracket is m. The ceiling of the power plant is vibrated with an amplitude of X_0 m at frequency of ω rad/s causing the fixture to oscillate as shown in Figure Q1(ii).

(a) Show that the linearized differential equation of motion of the system for a small amplitude of oscillation in terms of coordinate x is given as

$$\ddot{x} + \omega_n^2 x = \omega_n^2 X_0 \sin \omega t$$

where $\omega_n = \sqrt{g/l}$ and x is measured from the system equilibrium position.

[10 marks]

(b) Show that the vertical distance h from the mass m of the bulb and its bracket to the node of the cable is given by the relation $h = l(\omega_n/\omega)^2$.
(Hint: the node will occur when x_0 and x are opposite to each other)

[10 marks]

(c) Show that the node is at the midpoint of the cable when $\omega = \sqrt{2}\omega_n$.

[5 marks]

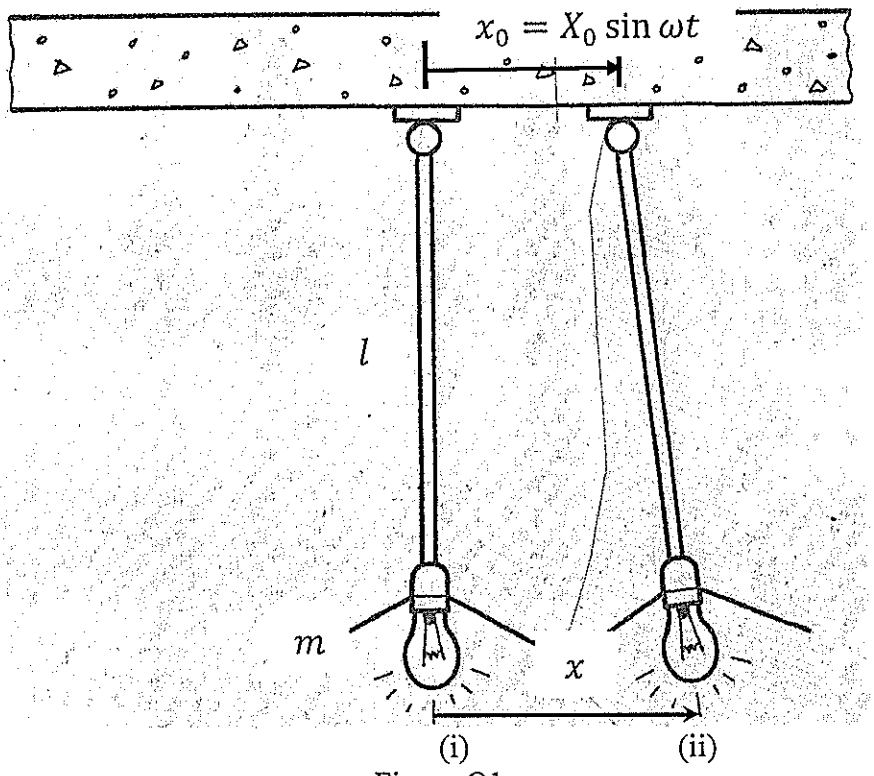


Figure Q1

Continued .....

Question 2

(a) Figure Q2(a) shows a compressor with mass m of $m = 906$ kg and unbalance of 0.25 kg-m, and is operated at 1800 rpm. At this speed, undesirable vibration occurs when the compressor is attached directly to the floor of a small building in a manufacturing plant. To reduce the annoying and potentially damaging vibration on the floor, it is proposed to isolate the compressor by mounting it on a square concrete block which is separated from the rest of the floor. The block is resting on a silty-clay soil. The gap between the block and the floor is filled with asphalt filler. The density ρ of the concrete of the block has been measured as $\rho = 23000\text{N/m}^3$, and the coefficient k_c of the silty-clay soil has been experimentally determined to be $k_c = 5 \times 10^6\ \text{N/m}^3$. The geometry of the compressor leads to choosing a 2×2 m block. Determine the depth h that will yield a 75 % reduction in the force transmitted by the compressor block system to the supporting soil. What is the maximum total settling of the soil?

[13 marks]

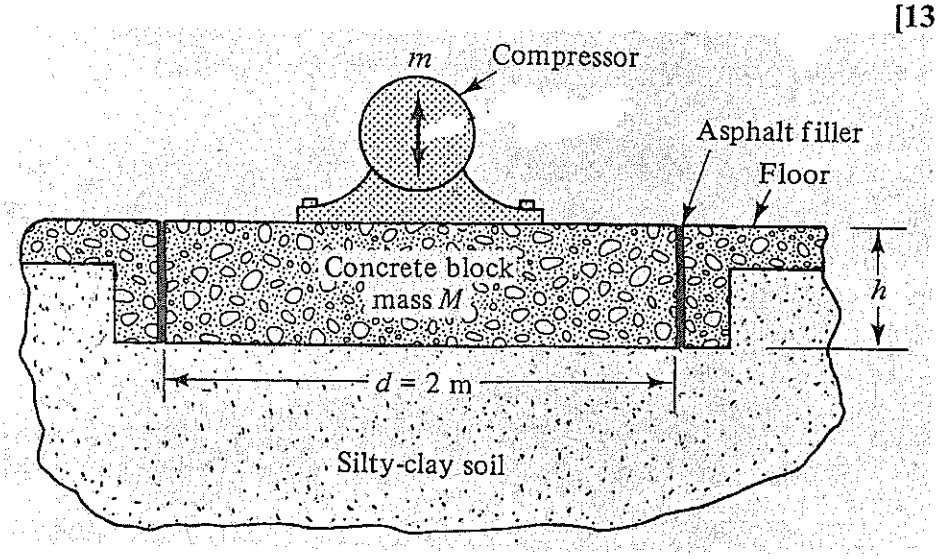


Figure Q2(a)

(b) A compressor has a mass of 30 kg. The compressor is mounted on an isolation pad in order to isolate the foundation from the vibration due to the compressor. From the experimental data, it is found that the stiffness of the pad is 6×10^5 N/m. If the compressor is operated at the frequency of 100 rad/s and the magnitude of the excitation is observed to be 350 N, determine the damping ratio of the isolator and its maximum deflection due to this excitation. Given that the phase difference between the excitation and steady state response is measured to be 24.3^0.

[12 marks]

Continued .....

Question 3

A uniform bar of length l and centroidal mass moment of inertia I is attached to a torsional spring of stiffness k_1, and then is subjected to the force $F = F_0 \sin \omega t$ at its end as shown in Figure Q3. The spring of stiffness k_2 and the mass m_2 are attached to the right end of the bar to act as a vibration absorber in eliminating large angular displacement θ of the bar when $\omega = \sqrt{k_1/I}$.

(a) Derive the equations of motion of the system in terms of θ and x which are the angular displacements of the bar and the displacement of the mass m_2, respectively.
[10 marks]

(b) Determine the steady-state response of the system.
[10 marks]

(c) Find a relationship between the frequency ω of the applied force and the values of m_2 and k_2 of the absorber that will make $\theta = 0$.
[5 marks]

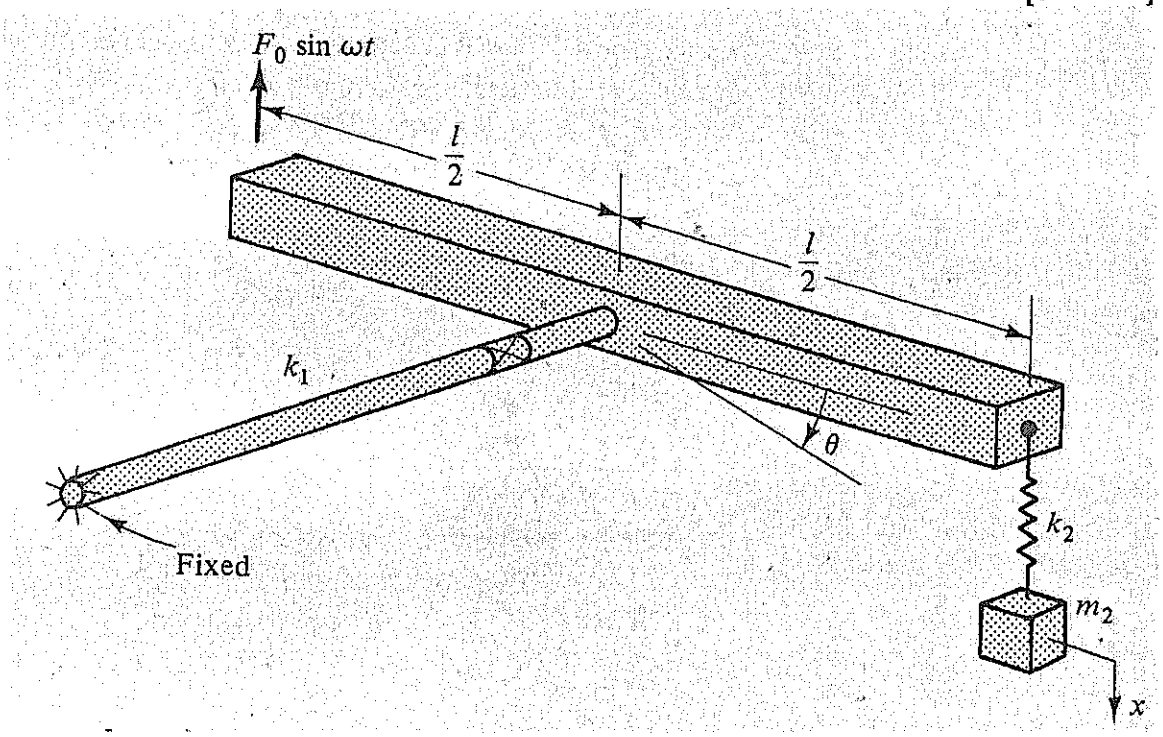


Figure Q3

Continued .....

Question 4

Figure Q4 shows a package with mass m which is attached to a rigid massless plate P through an elastic steel cable. The cable has a length l and diameter d. The Young's modulus of the cable is $E = 200 \times 10^9\ N/m^2$. The system is released from rest and fall freely through the height h before the plate strikes the rigid support. Assume that the plate does not rebound. If $m = 200\ kg$, $l = 10\ m$ and $d = 120\ mm$,

(a) determine the maximum height the system can be dropped such that the cable's elastic strength, σ_e , of $560 \times 10^6\ N/m^2$ is not exceeded.
(Hint: maximum stress, σ_{max} = maximum tension in cable/area, $\sigma_{max} \leq \sigma_e$)
[17 marks]

(b) If the system is dropped from the height as determined in part (a), determine the time t_0 at which the stress in the cable is zero and the displacement of the package at this time.
[8 marks]

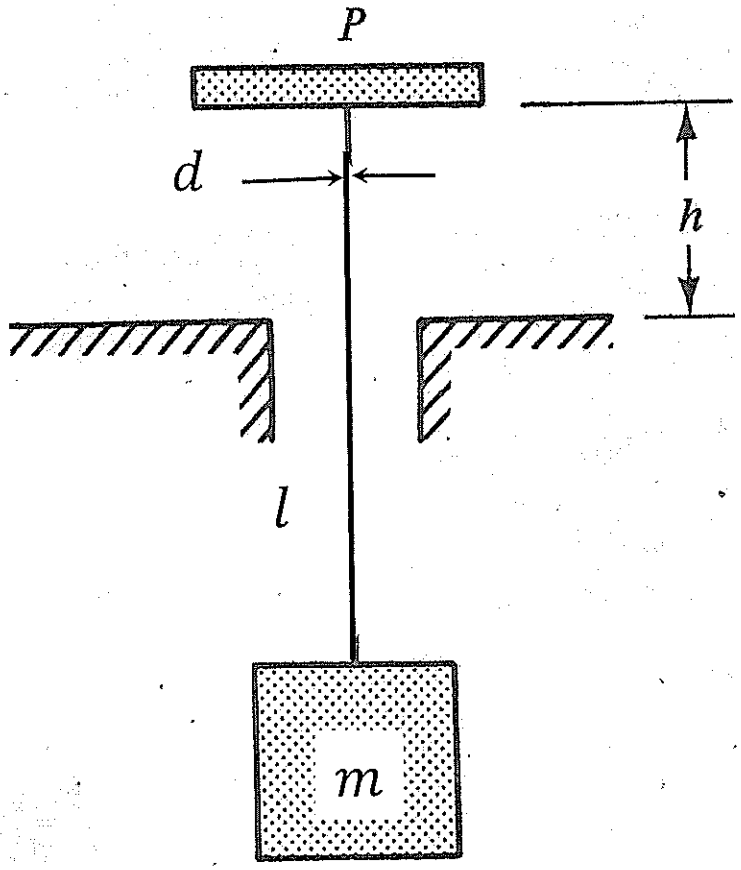


Figure Q4

Continued .....

Question 5

Figure Q5 shows an astronaut, who is buckled in the crew capsule of a rocket, is cushioned against vibration. The astronaut has mass of m. After the firing of the final stage of the rocket, the acceleration of the capsule reduces slowly according to $\ddot{u} = ae^{-bt}$, where a and b are constants.

(a) Derive the equation of motion of the system in terms of z where $z = x - u$ that is z is the relative displacement of the astronaut x with respect to the capsule u.

[5 marks]

(b) Determine an expression for the acceleration $\ddot{x}$ of the astronaut feels, if initial conditions are given as $z(0) = 0$ and $\dot{z}(0) = 0$.

[15 marks]

(c) If at steady-state the acceleration that the astronaut can withstand without losing consciousness is at most 5g, find the maximum acceleration constant, a, of the rocket that can be used.

[5 marks]

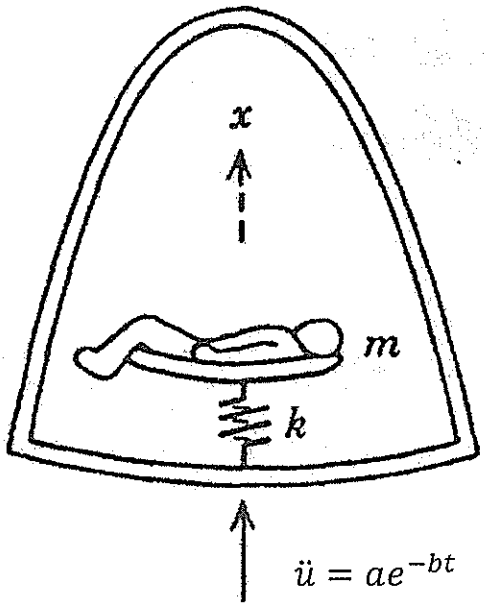


Figure Q5

Continued .....

Appendix 1

Vibrations

Equations of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D \sin(\omega t - \varphi)$$

$$D = \frac{F_0/k}{\sqrt{[1-r^2]^2 + (2\zeta r)^2}} \qquad where \quad r = \frac{\omega}{\omega_n}$$

$$\varphi = \tan^{-1} \frac{2\zeta r}{1-r^2}$$

$$x_c(t) = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} \qquad \zeta > 1$$

$$x_c(t) = (B + Ct) e^{-\omega_n t} \qquad \zeta = 1$$

$$x_c(t) = e^{-\zeta \omega_n t} (B \cos \omega_d t + C \sin \omega_d t) \qquad \zeta < 1$$

$$\omega_n = \sqrt{k/m} = 2\pi f_n, \qquad f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\lambda_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \qquad \zeta \geq 1$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \qquad \zeta < 1$$

Force and Displacement Transmissibility

$$\frac{F_T}{F_0} = \frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}}$$

Rotating Unbalance: Amplitude of Response

$$X = \frac{m_0 e}{m + m_0} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Fourier's series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

$$a_n = \frac{2}{\tau}\int_0^{\tau} x(t) \cos n\omega t \, dt \qquad b_n = \frac{2}{\tau}\int_0^{\tau} x(t) \sin n\omega t \, dt$$

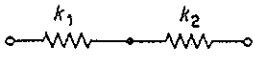
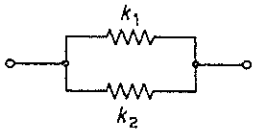
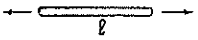
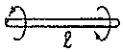
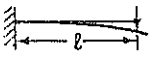
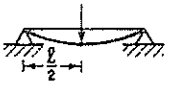
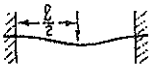
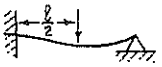
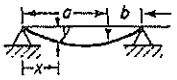
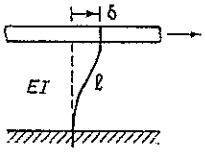
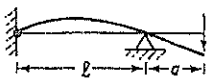
Unit response function (underdamped)

$$h(t-\tau) = \frac{e^{-\xi\omega_n(t-\tau)}}{m\omega_d} \sin \omega_d (t-\tau)$$

Continued .....

Appendix 2

Table of Spring Stiffness

	$k = \frac{1}{1/k_1 + 1/k_2}$	
	$k = k_1 + k_2$	
	$k = \frac{EI}{l}$	I = moment of inertia of cross-sectional area l = total length
	$k = \frac{EA}{l}$	A = cross-sectional area
	$k = \frac{GJ}{l}$	J = torsion constant of cross section
	$k = \frac{Gd^4}{64nR^3}$	n = number of turns
	$k = \frac{3EI}{l^3}$	k at position of load
	$k = \frac{48EI}{l^3}$	
	$k = \frac{192EI}{l^3}$	
	$k = \frac{768EI}{7l^3}$	
	$k = \frac{3EIl}{a^2b^2}$	$y_x = \frac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \frac{12EI}{l^3}$	
	$k = \frac{3EI}{(l + a)a^2}$	
	$k = \frac{24EI}{a^2(3l + 8a)}$	

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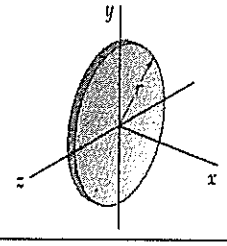
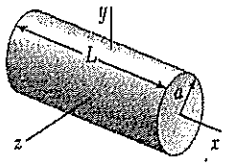
Appendix 3: Mass Moment of Inertia of Homogenous Solids



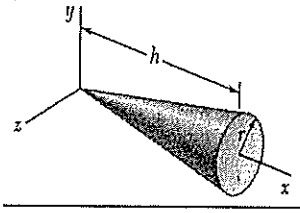
$I_z = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$



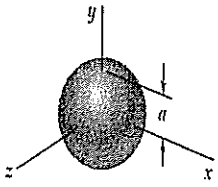
$I_z = \frac{1}{2}ma^2$

$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$

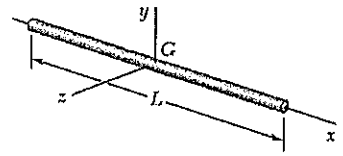


$I_z = \frac{3}{10}ma^2$

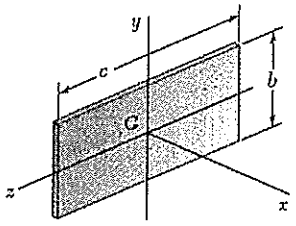
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



$I_x = I_y = I_z = \frac{2}{5}ma^2$



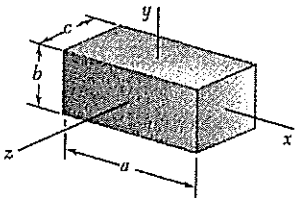
$I_y = I_z = \frac{1}{12}mL^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}mc^2$

$I_z = \frac{1}{12}mb^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}m(c^2 + a^2)$

$I_z = \frac{1}{12}m(a^2 + b^2)$

End of Paper